A New Class of Broad-Band Microwave 90-Degree Phase Shifters*

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Summary—In the type of circuits considered here, the input power is divided equally between two channels whose outputs are caused to have a very nearly 90° phase difference over a broad frequency range. Networks suitable for application at low frequencies which perform the above function have been widely investigated.[1-9] This report describes a new type of 90° differential phase shifter which has a constant resistance input, and which is useful over bandwidths as large as 5:1 in the microwave region.

Theory

Basic Phase-Shifting Element

THE circuits to be described employ sections of coupled-strip transmission lines operating in the TEM mode as key elements. One such coupled-strip transmission-line phase-shift element is shown in Fig. 1. Two parallel-coupled lines of equal length are connected at one end; ideally this connection should be of zero length. The unconnected ends serve as the input and output of a two-terminal-pair network. The frequency behavior of a coupled-line network connected in this manner, and also that of other related coupled-line circuits, has been derived by Jones and Bolljahn.[10]

The equations for the image impedance Z_I, and phase constant, ϕ, of coupled lines connected as shown in Fig. 1 are, in terms of the even- and odd-mode *characteristic* impedances of the lines and their length,

$$Z_I = \sqrt{Z_{0o}Z_{0e}}, \tag{1}$$

and

$$\cos\phi = \frac{\dfrac{Z_{0e}}{Z_{0o}} - \tan^2\theta}{\dfrac{Z_{0e}}{Z_{0o}} + \tan^2\theta}, \tag{2}$$

where

Z_{0e} is the characteristic impedance of one line to ground when equal in-phase currents flow in both lines,

Z_{0o} is the characteristic impedance of one line to ground when equal out-of-phase currents flow in both lines.

$\theta = \beta l$ is the electrical length of a uniform line of length l and phase constant β.

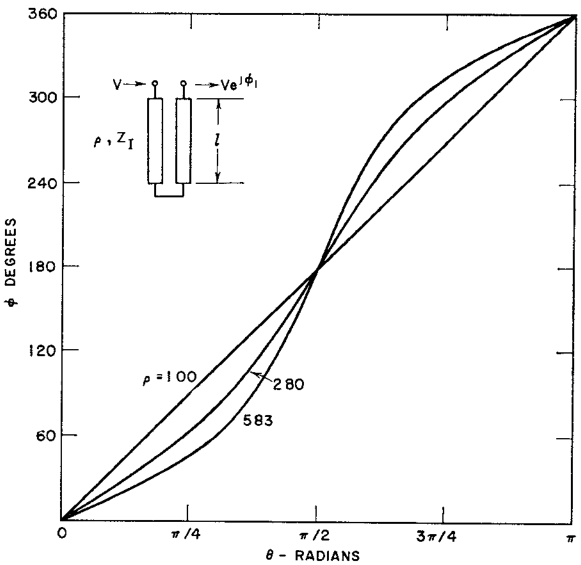

Fig. 1—Coupled-transmission-line element with ends connected and curves of its phase response for three values of ρ.

From (1) Z_I is seen to be a constant independent of frequency. The phase-shift function, ϕ, of such a coupled-line element is plotted against θ, the frequency variable, in Fig. 1 for three values of ρ, where ρ is defined as $\rho = Z_{0e}/Z_{0o}$. Such a network element possesses a sufficient number of independent design parameters to permit its use in a variety of phase-shift networks. This can be demonstrated as follows.

From the orthogonality relations that define them,[11] it is seen that Z_{0o} and Z_{0e} are independent quantities. Hence, the product $(Z_{0o}Z_{0e})$ and the ratio $\rho = Z_{0e}/Z_{0o}$ can be independently specified. It then follows, using (1) and (2), that the image impedance of the coupled-line network can be chosen independently of its image phase constant, ϕ. Therefore, provided that ϕ can be properly specified by suitable choices of ρ and line length l, and that a power divider can be designed that

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is matched at all frequencies, it follows that a network with a desirable differential-phase response can be obtained by connecting in parallel a coupled-line network and a suitable length of uniform transmission line. Such a network is described below.

Type-A Network

The most elementary form of such a network, termed a Type-A network, is shown schematically in Fig. 2, together with a plot of phase shifts, ϕ_1 through the coupled portion, and ϕ_2 through the uniform portion. The characteristic impedance of the length of uniform line is $Z_0 = Z_I$, and the outputs of both branches are assumed to be matched. The input impedance of the network is $Z_I/2$, a constant independent of frequency.

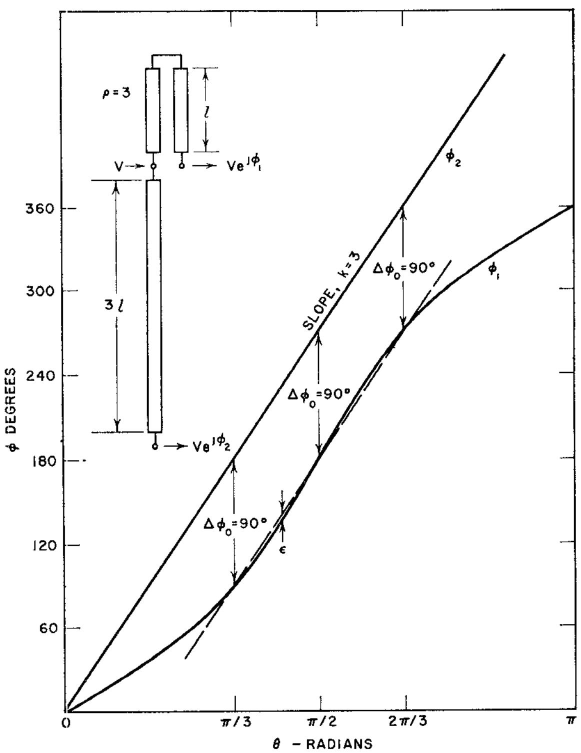


Fig. 2—A Type-A network and curves of phase response for each of its two branches.

The phase shift through the uniform transmission line, ϕ_2, is represented by the straight line of slope k through the origin, where k is the ratio of the length of the uniform line to that of the coupled lines. Inspection of (2) for phase shift through the coupled-line portion indicates that ϕ_1 has odd symmetry about $\theta = n\pi/2$. Thus a dashed line, also of slope k, drawn through the point $(\theta = \pi/2, \phi = \pi)$ as shown in Fig. 2, intersects the ϕ_1 curve in two other points equal distances from $\theta = \pi/2$. It now can be seen that the output phase difference, $\Delta\phi = \phi_2 - \phi_1$, can be made equal to 90° for three desired values of θ by means of (2). For the case illustrated in Fig. 2, $k = 3$, $\rho = 3.00$, and $\Delta\phi = 90°$ at $\theta = \pi/3$, $\pi/2$, and $2\pi/3$. The output phase difference at these three values of θ will be called $\Delta\phi_0$. At other points in the interval $\pi/3 < \theta < 2\pi/3$ and for a small distance outside this interval, the phase difference, $\Delta\phi$ will vary from $\Delta\phi_0$ by some small amount which is the phase error. A graph of the theoretical differential phase shift through the Type-A network of Fig. 2 is shown in Fig. 3. The differential phase shift is $90 \pm 4.8°$ over a 2.34:1 bandwidth, as shown. Other values of ρ yield different values of maximum phase errors and bandwidths for the Type-A network. For example, $\rho = 2.7$ yields a differential phase shift of $90 \pm 2.5°$ over a 1.95:1 bandwidth.

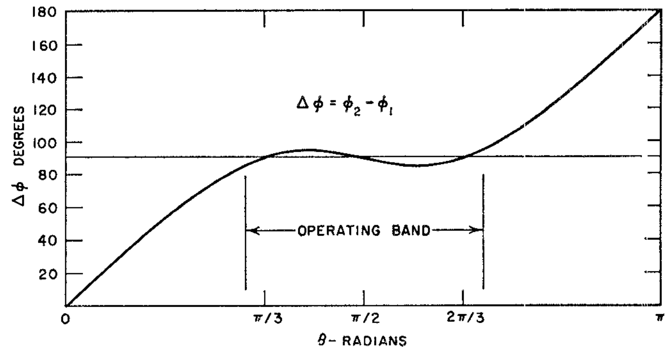


Fig. 3—Differential phase response of the Type-A network shown in Fig. 2.

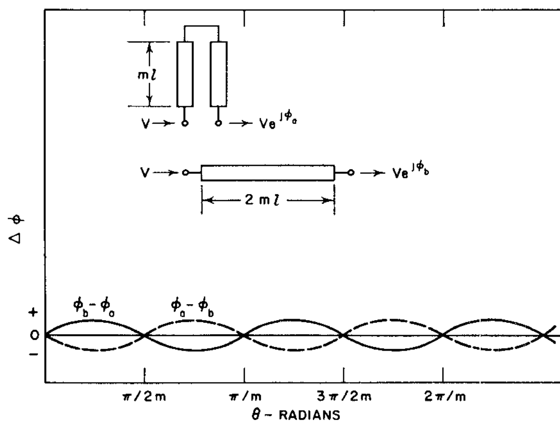


Fig. 4—Error-correcting network and its two possible differential phase responses.

Phase-Error-Correcting Network

It is possible to reduce the maximum phase error of a Type-A differential-phase-shift network by connecting another differential-phase-shift network in tandem. This second network is so designed that $\Delta\phi_0 = 0$ and its $\Delta\phi$ vs θ curve is approximately the negative of the error curve of the Type-A network in the band of interest. Such a network is shown in Fig. 4. It consists of two entirely separate parts, a section of uniform transmission line of length $2ml$ and a coupled-line section of length ml. It is possible, therefore, to connect the error-correcting network to the Type-A network in two ways. Consequently, the former has two $\Delta\phi$ vs θ curves which are the negatives of each other, as shown in Fig. 4. In a given case the parameters m and ρ of the error-correcting network, and the method of connecting the latter to the Type-A network must be chosen so that the net phase error in a given bandwidth is minimized.

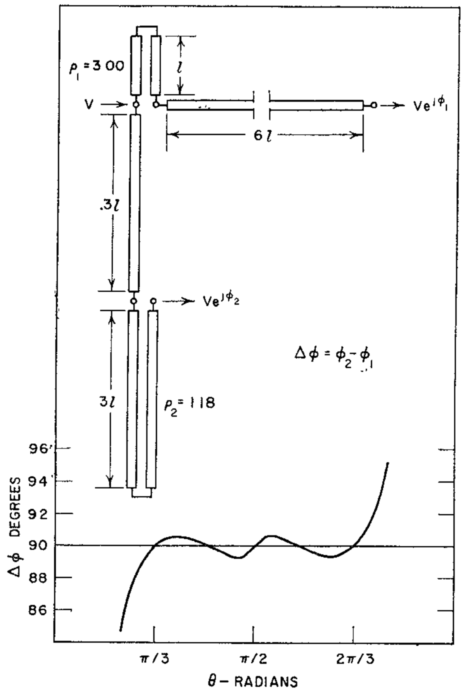


Fig. 5—A Type-B network and its differential phase response.

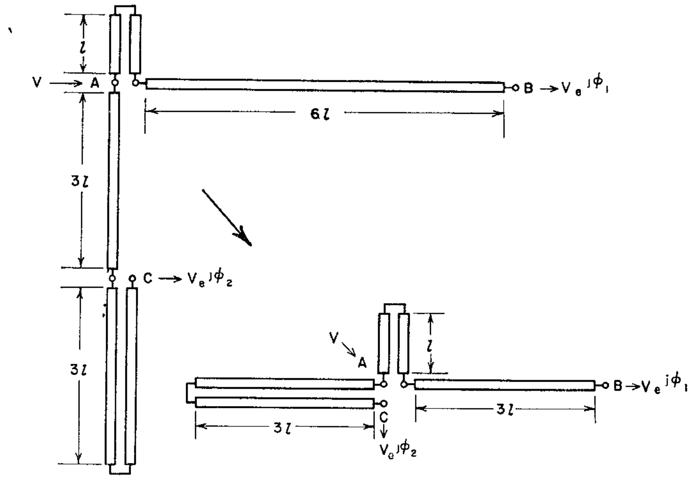


Fig. 6—Type-B network before and after removing equal lengths of uniform transmission line.

Type-B Network

A differential phase shifter that was designed as outlined above is shown in Fig. 5. In this case, m is set equal to 3. (A nonintegral value of m would destroy symmetry and almost certainly reduce the bandwidth of a given network configuration.) Thus, the error-correcting network differential-phase response is zero at $\theta=\pi/3$, $\pi/2$, and $2\pi/3$; and the resulting combination called a Type-B network is suitable for an approximately 2:1 bandwidth. Equal-ripple response may then be obtained in the band of interest with two additional points of zero phase error by properly choosing ρ_1 and ρ_2. The subscript 1 applies to the differential-phase-shifting coupled-line portion and the subscript 2 applies to the error-correcting coupled-line portion. Here again it is possible to choose various combinations of these parameters to obtain slightly different bandwidths and maximum phase errors. A natural choice is to have zero phase error at $\theta=\pi/3$ and at $\theta=2\pi/3$ as in the Type-A network. Since the error-correcting curve goes through zero at $\theta=\pi/3$ and at $\theta=2\pi/3$, $\Delta\phi_0$ of the differential-phase-shifting element is made equal to 90° at these points. Therefore, $\rho_1=3.00$ as in the Type-A network described in the preceding section. By choosing a point midway between bandedge and bandcenter for complete error cancellation, ρ_2 is found to be 1.18 by trial and error, and an almost equal-ripple response is obtained. The maximum phase error is thus 0.7° over a 2.13:1 bandwidth. The theoretical differential-phase response of this Type-B network is plotted in Fig. 5.

By shifting the points of zero phase error from $\theta=\pi/3$ and $\theta=2\pi/3$ and allowing a maximum phase error of 1.2°, the bandwidth can be extended to 2.32:1. For such a network, $\rho_1=4.10$, and $\rho_2=1.24$. Still greater bandwidths with resulting larger phase errors are possible with this type of network. Conversely, the maximum phase error can be reduced well below the 0.7° maximum of the first example, if a reduced bandwidth is acceptable, although such configurations have not been investigated.

Composite networks as described above often can be made more compact by removing equal lengths of uniform transmission line from each branch, without affecting the differential-phase response. Such a reduction in size is illustrated in Fig. 6, for the Type-B network.

Type-C Network

By reversing the connection of the error-correcting network, a third type, the Type-C network, is obtained. In this case, putting $m=2$ secures error cancellation over a broader frequency band than does the Type-B network and preserves the symmetry of the response. For the error-correcting network, $\Delta\phi_0=0$ at $\theta=\pi/4$ and at $\theta=3\pi/4$. By arbitrarily setting $\Delta\phi_0=90°$ for the differential-phase-shift network at $\theta=\pi/4$ and at $3\pi/4$, we find by means of (2) that $\rho_1=5.83$. Three values of ρ_2 were tried to minimize the phase error over as broad a band as possible. A value of $\rho_2=2.35$ yielded a 90° differential phase shifter with a ±5° error over a 5:1 band. Such a Type-C network and its theoretical $\Delta\phi$ vs θ curve are shown in Fig. 7. Further improvement in the theoretical performance of this type of network may be obtained by changing the value of ρ_1 a small amount and then optimizing the value of ρ_2.

Type-D and Type-E Netwoiks

Other configurations of differential-phase-shift networks employing coupled lines have been investigated. Type D and its derived Type E are shown in Fig. 8. In these types, the band center is at $\theta=\pi$, instead of at $\theta=\pi/2$ as in Types A, B, and C. A Type-E network having $\rho_1=3.0$ and $\rho_2=1.37$ yields a maximum phase error

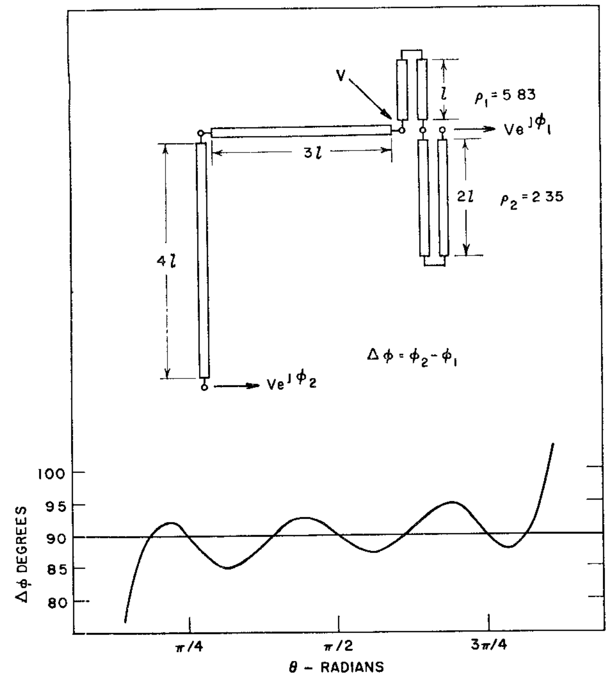
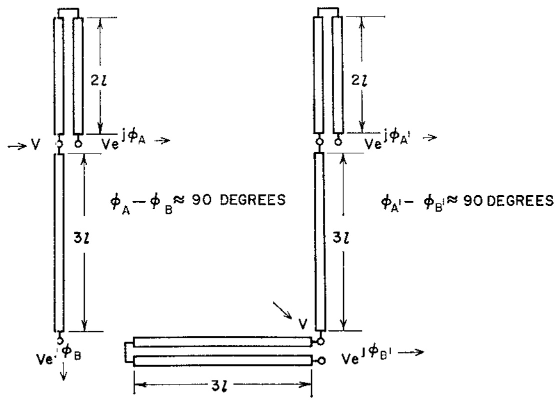


Fig. 7—A Type-C network and its differential phase response.



(a) (b)

Fig. 8—(a) Type-D network. (b) Type-E network.

of the order of 2° for a 2:1 band. Since this performance is poorer than that of the Type-B network which is its counterpart, Types D and E will not be discussed further.

Type-F Network

A more complex phase-shifting element is illustrated in Fig. 9. It consists of a coupled-line section with two degrees of coupling along different portions of its length. If the even- and odd-mode characteristic impedances are so chosen that

$$\frac{Z_{0e_1}}{Z_{0e_2}} = \frac{Z_{0o_2}}{Z_{0o_1}}, \tag{3}$$

the image impedance is

$$Z_I = \sqrt{Z_{0e_1}Z_{0o_1}} = \sqrt{Z_{0e_2}Z_{0o_2}},$$

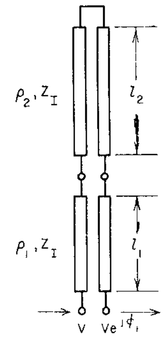


Fig. 9—A more complex type of phase-shifting element.

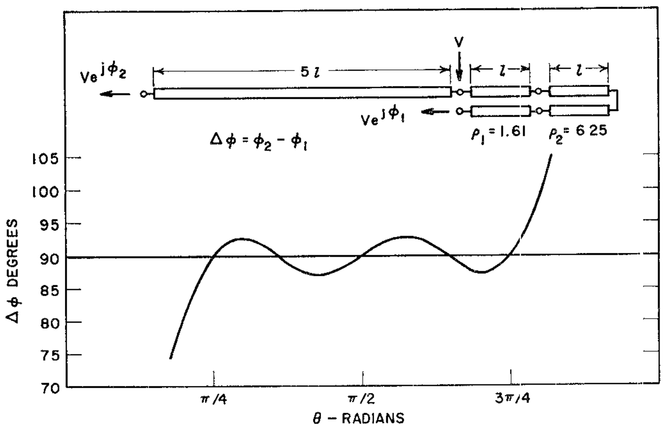


Fig. 10—A Type-F network and its differential phase response.

and the phase shift, ϕ, is

$$\phi_1 = \cos^{-1}\frac{\rho_1 - \tan^2\left(\tan^{-1}\left[\frac{Z_{0o_2}}{Z_{0o_1}}\tan\theta_2\right] + \theta_1\right)}{\rho_1 + \tan^2\left(\tan^{-1}\left[\frac{Z_{0o_2}}{Z_{0o_1}}\tan\theta_2\right] + \theta_1\right)}. \tag{4}$$

Here the subscripts 1 and 2 refer to the two sections of the coupled transmission line as shown in Fig. 9. A differential phase shifter employing this element, designated Type F, is shown in Fig. 10. The length of the uniform transmission line is $5l$. The two portions of the coupled lines with different degrees of coupling are each of length l. This equality of lengths is necessary in order to obtain a differential-phase response that is symmetrical about the center frequency. Such a symmetrical output is desired because it is reasonable to expect that the useful bandwidth of the network is thereby maximized. A consequence of making the two coupled portions of equal length, however, is that $\Delta\phi_0 = 90°$ at $\theta = \pi/4$ and at $\theta = 3\pi/4$. This network is suitable for bandwidths of about 3:1, for this reason.

Three sets of values of the parameters ρ_1 and ρ_2 for the two coupled portions have been tried. The values $\rho_1 = 1.612$ and $\rho_2 = 6.25$ yield a 3.24:1 bandwidth with a phase error of $\pm 2.8°$ as shown in Fig. 10.

Comparison of the Various Networks

A summary of the differential-phase-shift network types and their approximate characteristics is given below in Table I.

TABLE I
NETWORK TYPES AND THEIR CHARACTERISTICS

Type	Ratio-Upper to Lower Frequency	Maximum Theoretical Phase Error	Remarks
A	2.0	±2.8°	Simplest and most compact design (Fig. 2)
B	2.0–2.5	±0.7°–2.0°	(Fig. 5 and Fig. 6)
C	5.0	±5.0	Bulky, most sensitive to mechanical tolerances (Fig. 7)
D, E	—	Excessive	Not recommended (Fig. 8)
F	3.2	±2.8°	Simpler and more compact than Types B and C (Fig. 10)

Note: the above values of bandwidth and phase error are typical of each type of network and are not necessarily optimum.

EXPERIMENTAL MODEL

Design

A Type-C network was constructed in strip-line for the 300–1500 mc band, to test the foregoing theory. The layout of the circuit is shown in Fig. 11. The characteristic impedance of each branch is 50 ohms; the input impedance at the T junction is 25 ohms, therefore, but no attempt was made to match the input since the differential phase shift is not affected by such a mismatch provided the tee is symmetrical. Symmetry was maintained to the extent that this was possible in the remainder of the network.

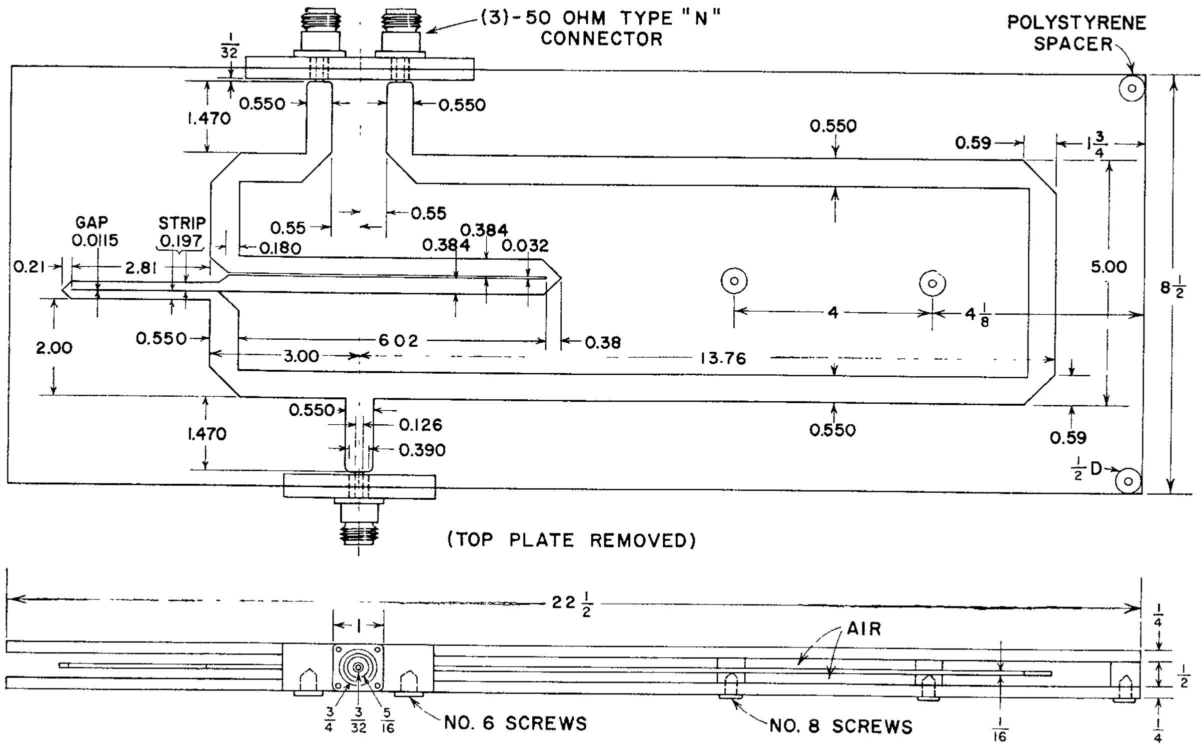


Fig. 11—Experimental Type-C phase shifter for the 300–1500-mc band. *Note:* dimensions in inches; all corner bevels 45°.

The ground-plane spacing was chosen as one-half inch and the thickness of the center conductor as 1/16 inch. The dielectric is air, except for some polyfoam strips supporting the center conductor. Four polystyrene cylinders and the two connector blocks separate the ground planes. The width of the center strip of the 50-ohm characteristic-impedance portion is obtained from design curves for strip transmission lines[12] and the widths and spacings of the coupled-strip portions of the network are obtained from coupled-strip transmission-line design formulas[11] developed at Stanford Research Institute by S. B. Cohn.

[12] S. B. Cohn, "Characteristics impedance of the shielded-strip transmission line," IRE TRANS., vol. MTT-2, pp. 52–57; July, 1954.

The three right angle transmission-line bends in each branch were mitered in a manner known to reduce reflections to a very low value. The junctions between the transmission line and phase shifting coupled-line elements were similarly mitered, as were the short lengths of line which connect each pair of coupled lines.

In calculating the required lengths of uniform transmission line and coupled lines the following simplifying assumptions were made: 1) the uniform line and the right-angle bends in each branch behave as uniform lines of different length; 2) the short lines which connect the coupled lines are uniform 50-ohm lines whose lengths can be found by measuring along their centerlines. A simple analysis then showed that if assumption 2) is correct, these connecting lines produce a phase shift at bandcenter which is proportional to their lengths multiplied by $\sqrt{\rho}$. All lengths were then calculated for exactly 90° phase difference at bandcenter, which is 900 mc.

Results

The differential-phase output of the experimental network was measured by a substitution method. First tests showed a slightly rising average characteristic in the $\Delta\phi$ vs frequency curve as seen in Curve B of Fig. 12. This indicated either that the uniform transmission-line branch was too long or that the coupled portions were too short. However, the peaks of the experimental Curve B were displaced toward the higher frequencies, as compared with the theoretical Curve A, and this fact alone indicated that the coupled-line elements were made too short.

The experimental differential phase shifter was then adjusted to make the phase difference 90° on the average, as follows. A straight line was drawn through the 90° point on the ordinate of Fig. 12 so that Curve B oscillates about this line approximately uniformly. The slope of this line was then used to calculate the length by which the uniform-line branch of the phase shifter must be reduced in order to match the unwittingly shortened coupled-line portions. Thus the amount calculated is

$$\text{Slope} \times \text{wavelength} = \frac{(137° - 90°)}{360°} \times \frac{\text{velocity of light}}{1800 \text{ mc}} = 0.86 \text{ inch}.$$

The dimensions given in Fig. 11 reflect this adjustment and Curve D of Fig. 12 is the resultant differential phase

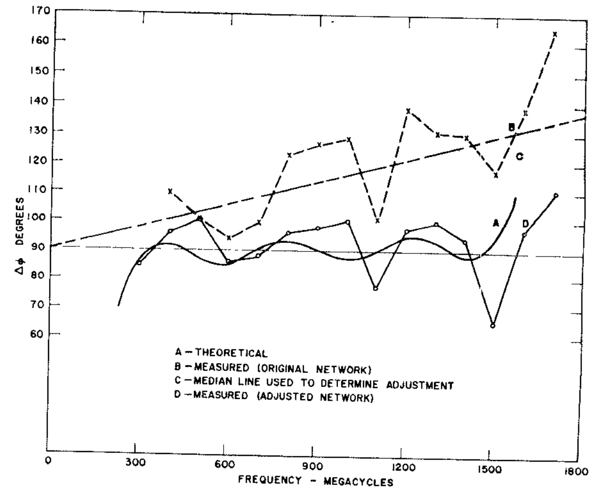


Fig. 12—Performance of a Type-C experimental phase shifter.

output. It is seen that the differential phase output oscillates about an average value of 90°, although the maximum phase error is larger than that predicted.

It is believed that the performance of the experimental phase shifter could be further improved by compensating for the discontinuities at the input to the coupled lines and at the connection between coupled lines.

Conclusion

The use of coupled transmission-line elements makes possible the design of broad-band, matched differential phase-shift networks for the microwave region. The techniques employed here in the design of 90° differential phase shifters also may be used to provide any other amount of differential phase shift over very broad frequency bands.

Acknowledgment

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